The Radar Range Equation

Short Course on Radar and Electronic Warfare

Kyle Davidson

Introduction

- The goal is understanding and applying the radar range equation

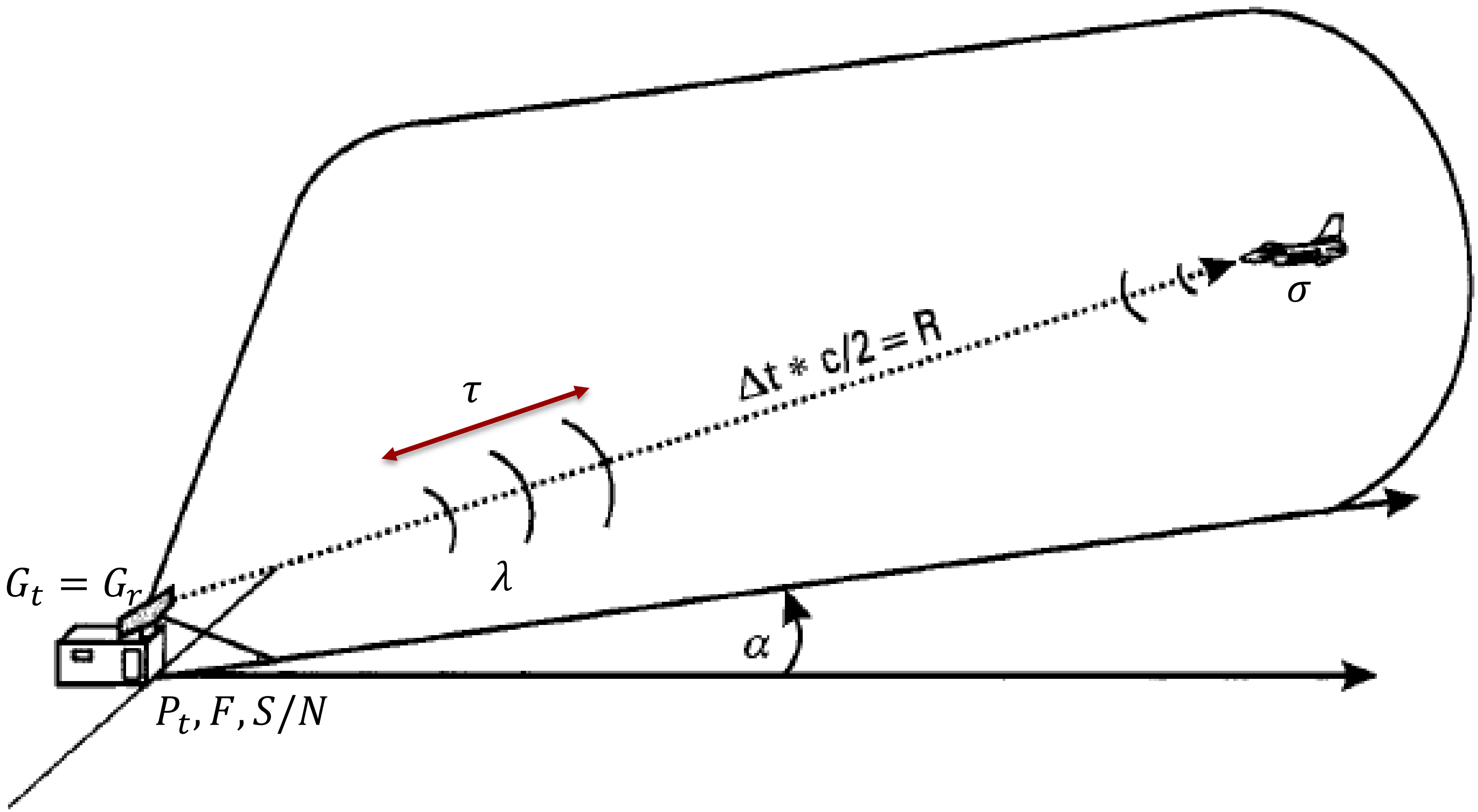
What is the radar range equation?

$$R^4 = \frac{P_t \tau G_t G_R \sigma \lambda^2}{(4\pi)^3 kTF(S/N)}$$

What does it mean?

- It expresses the relationship between the radar detection range and the radar and the target’s characteristics
- There are many forms, this is one of the most common
- Next building it…

Factors in the Range Equation

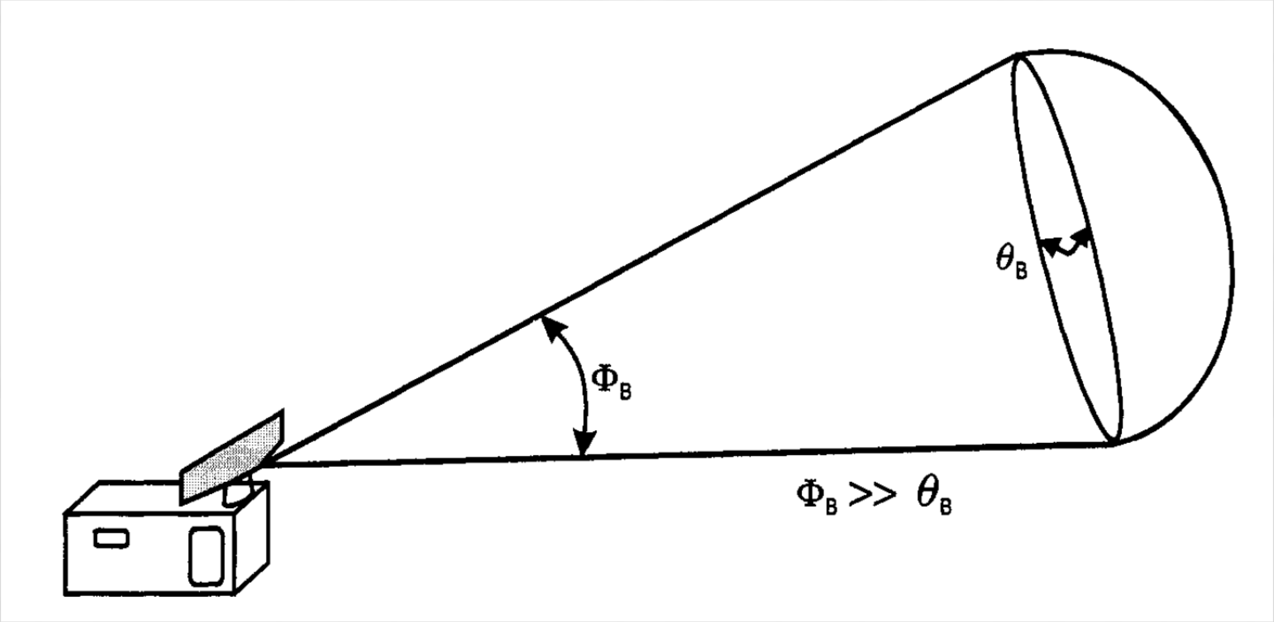


Round Trip Time

$$R = \frac{c\Delta t}{2}$$

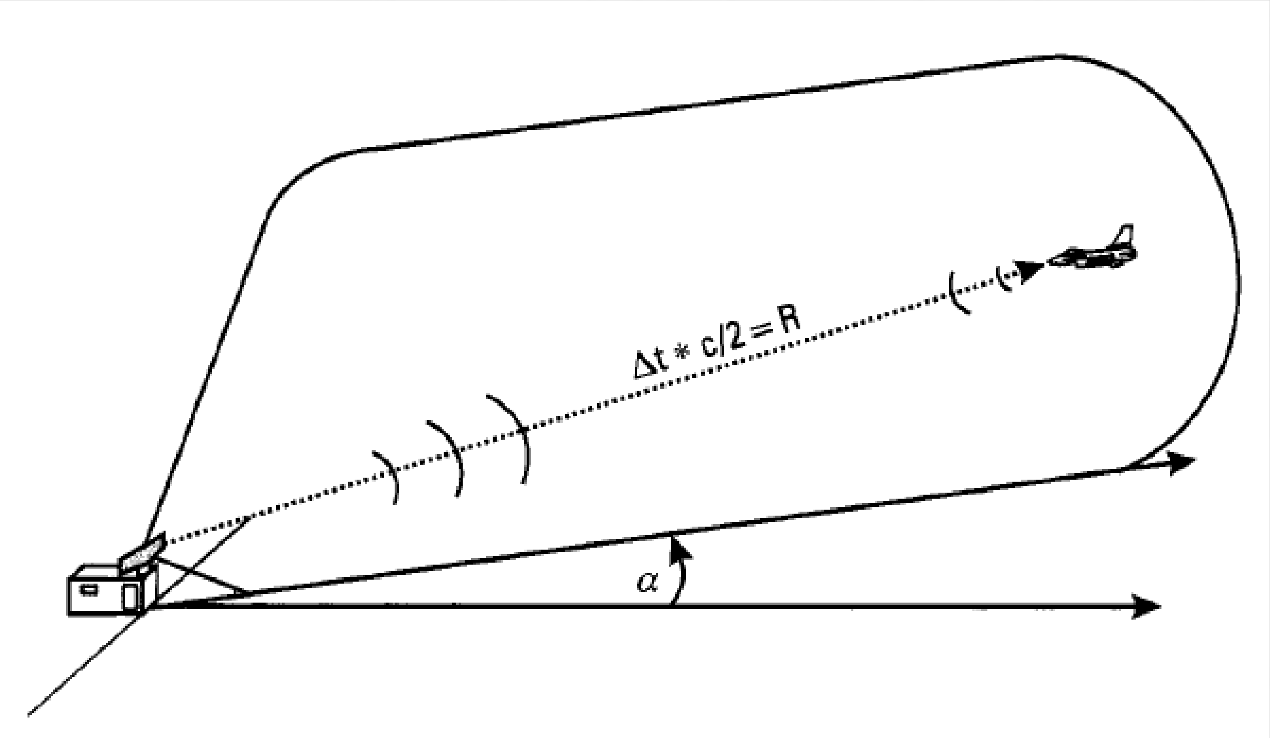
- R = range
- c = speed of light in a vacuum (3 x 10^8 m/s)
- Δt = round trip time

Power Density



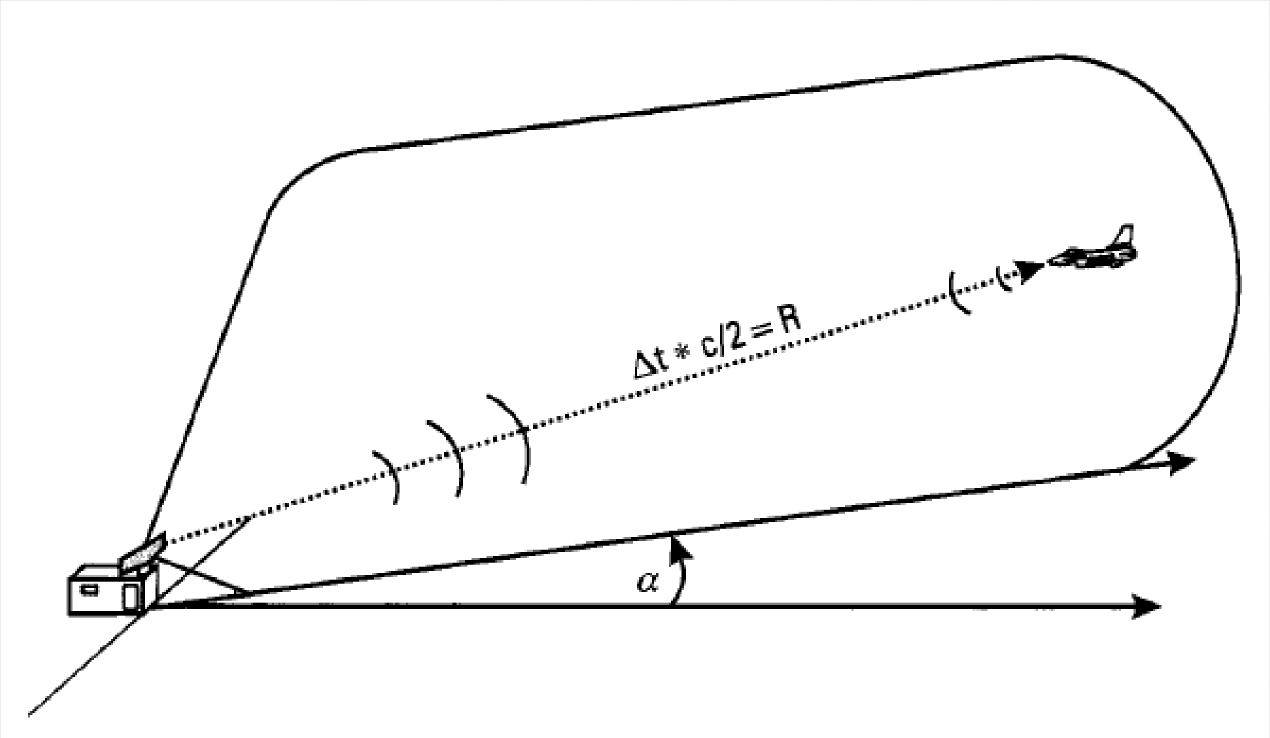
$$p = \frac{P_t G_t}{4\pi R^2}$$

Power Radiated from the Target



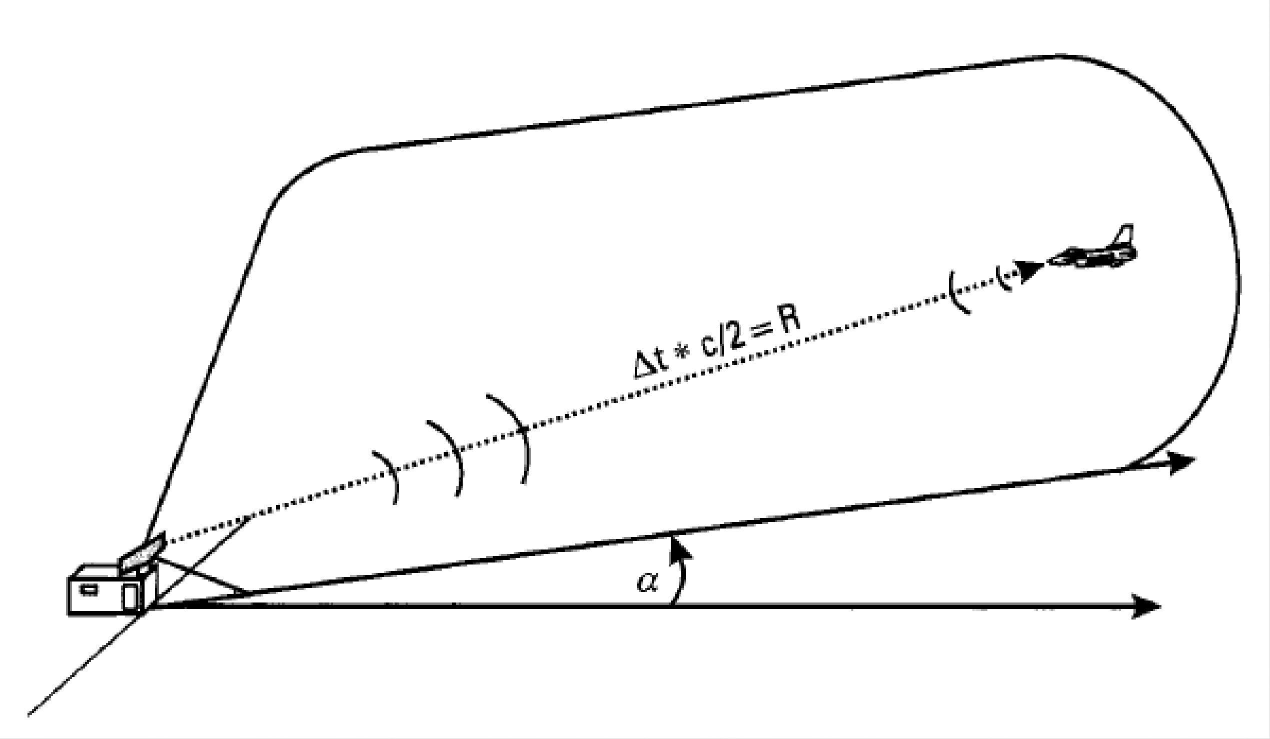
$$P_r = p\sigma = \frac{P_t G_t \sigma}{4\pi R^2}$$

Power Density at Radar Receiver



$$p_r = \frac{P_t G_t}{4\pi R^2} \sigma \frac{1}{4\pi R^2}$$

Received Signal Power



$$S = \frac{P_t G_t}{4\pi R^2} \sigma \frac{1}{4\pi R^2} A_{eff}$$

Simplifying the Received Power…

$$A_{eff} = \frac{G_r \lambda^2}{4\pi}$$

$$S = \frac{P_t G_t}{4\pi R^2} \sigma \frac{1}{4\pi R^2} \frac{G_r \lambda^2}{4\pi}$$

$$S = \frac{P_t G_t G_R \sigma \lambda^2}{(4\pi)^3 R^4}$$

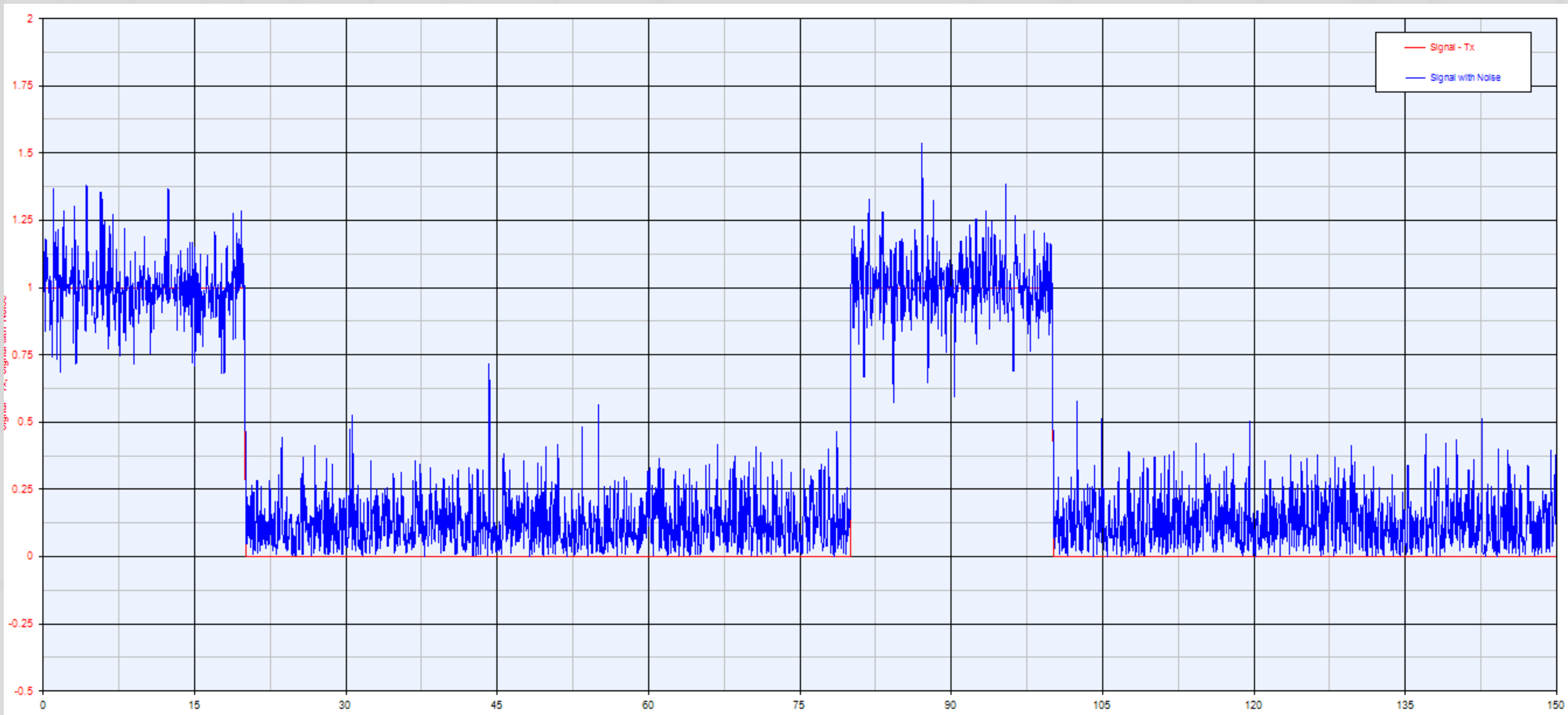
What about the Noise?

- The received noise power is:

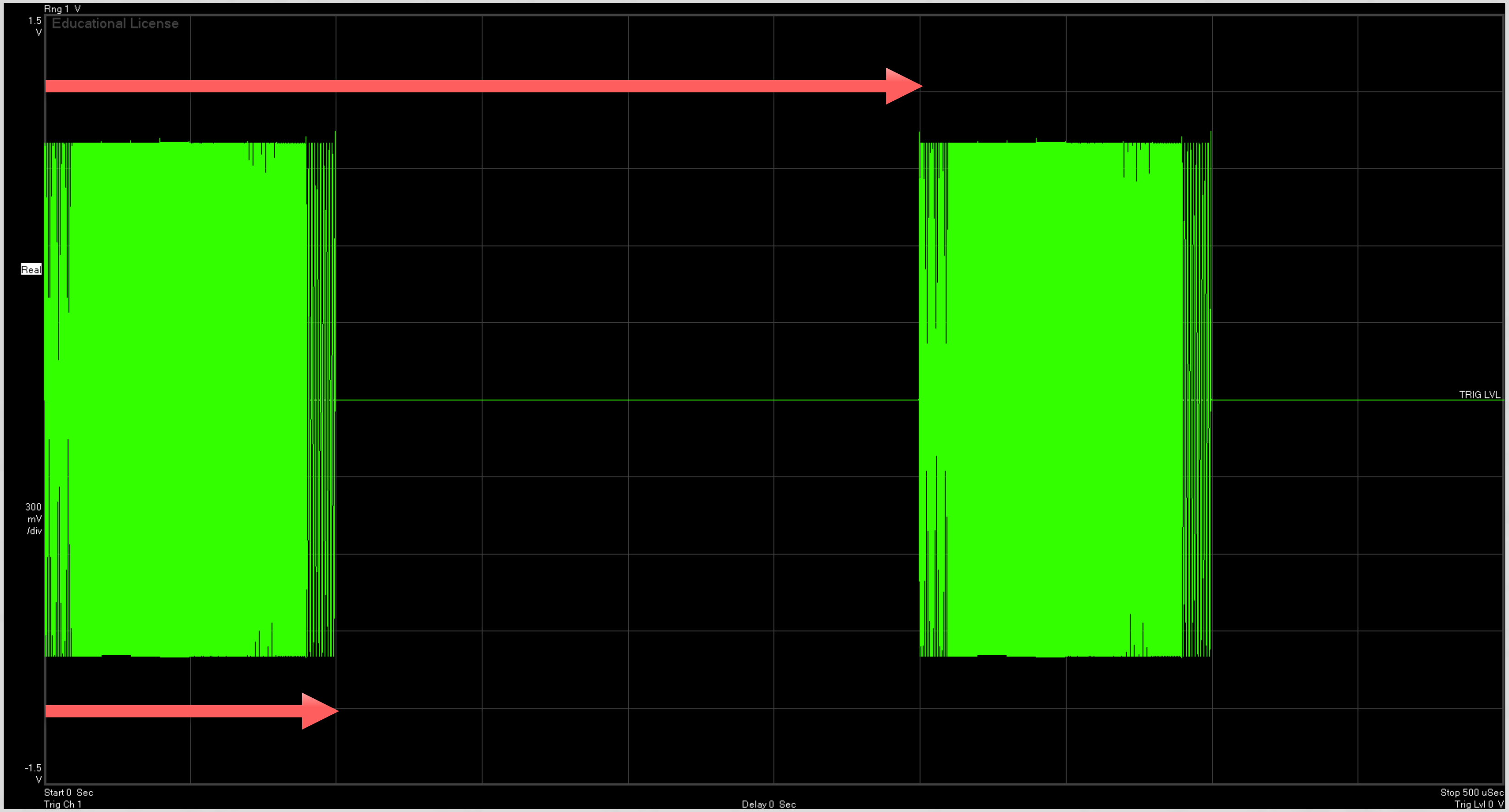
$$N = kTBF$$

- $k = 1.38 \times 10^{-23}\ W/(HzK)$
- T = 290 K
- B = receiver bandwidth
- F = noise figure

Effects of Noise (SNR = 10 dB)



Power vs. Energy



Signal to Noise Ratio

- Ratio of signal to noise power
- Strong indicator of ability to detect a radar return

$$(S/N) = \frac{S}{kTBF} = \frac{ST_{obs}}{kTF} = \frac{E}{kTF} = \frac{E}{N_0}$$

Received Signal Energy

- Taking into account the duty cycle to determine the average transmitted power
- The received energy is then:

$$E = \frac{P_t \tau G_t G_R \sigma \lambda^2}{(4\pi)^3 R^4}$$

The Complete RRE

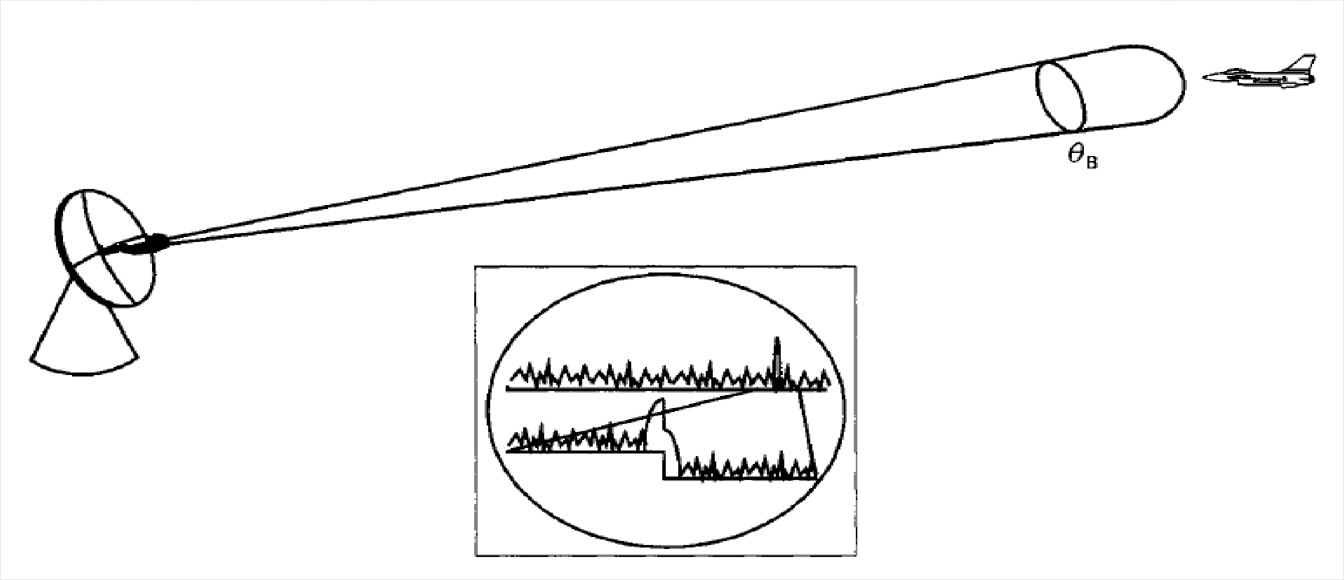
- Assuming a matched filter is used:

$$S/N = \frac{P_t \tau G_t G_R \sigma \lambda^2}{(4\pi)^3 kTFR^4}$$

- Then solving for the range:

$$R = \sqrt[4]{\frac{P_t \tau G_t G_R \sigma \lambda^2}{(4\pi)^3 kTF(S/N)}}$$

Incorporating the Antenna Scan



- Time on target

$$T_{ot} = \frac{\theta_B}{\omega_a}$$

- θ_B = antenna beam width
- ω_a = antenna rotation rate

Integrating the Pulses

- Coherent receivers

$$N_i = f_r T_{ot}$$

- $f_r = \text{PRF}$
- Non-coherent receivers use a $\sqrt{N_i}$ factor as an estimate of the effects of non-coherent integration

Integrating the Pulses and Losses

$$R = \sqrt[4]{\frac{N_i P_t \tau G_t G_R \sigma \lambda^2}{(4\pi)^3 kTF \left(\frac{S}{N}\right)_{P_d} L_i}}$$

- The loss factor incorporates losses from all aspects of the radar system:
  - Transmitter
  - Receiver
  - Radar channel
  - And more!

But, the Radar Horizon…

- The earth is curved!
- Range is limited by this curve, and the height of the radar and target above it

$$R_H \approx 4123 \times \sqrt{H_r}$$